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Article

Economic Analysis for Achieving Allocative-Technical Efficiency in Sustainable Agricultural Projects (Calf Fattening Farm as A Model)

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ABSTRACT

Livestock production projects are environmentally friendly, as they provide farmers with organic fertilizers and year-round job opportunities. This helps reduce the use of chemical fertilizers, thereby simultaneously achieving the goal of environmental improvement and poverty reduction. This research conducted an economic analysis of a calf fattening farm in Diyala Province for 2024. The research adopted a quantitative economic analysis methodology using a multivariate production function in the double logarithmic form. The research aimed to conduct a quantitative economic analysis to determine the optimal combination of production resources to achieve optimal production. Based on the prices of production resources, the cost equation was formulated, and then the Lagrange multiplier method was used to create the objective function. The optimal production reached (827) K.g/calf, which represents the weight of the calf during the fattening period of (25) weeks. The economic analysis showed that the elasticity of production resources was less than one, which means that the production process is in the second stage of the law of diminishing returns, where technical efficiency is achieved. Through the analysis of the allocative efficiency of production resources, it became clear that there is an inefficient use of dry fodder and green fodder, as the elasticity of these two resources was greater than one, and less than one for the other resources. The study recommended increasing the effective role of agricultural extension to educate calf farmers on the application of scientific research findings to achieve optimal production levels. It also recommended In addition to supporting fodder.

INTRODUCTION

Calf fattening involves converting excess food into meat, along with fat deposits. This results in a daily weight gain in the calf. In terms of economic importance, calf fattening projects rank second only to the broiler poultry industry, due to the increasing demand for red meat. In addition to achieving significant economic returns, they are also important in maintaining herd fertility and productivity [1]. Developing livestock is a national goal to meet the population's need for animal products. To achieve this goal, the calf farmer must have sufficient knowledge of the technical aspects of managing the calf herd at the calf farm during its various life stages [2]. Among the economic studies that examined calf-fattening projects, [3] conducted a study in Adamawa State- Nigeria. The researchers concluded that these projects help provide

job opportunities and improve farmers' income, but they also face some problems due to high interest rates and difficulty in securing capital [4]. conducted a study on buffalo calf fattening projects in Beheira Province - Egypt. Through statistical analysis of the production function, the researcher concluded that the majority of buffalo farmers in the study area were unable to achieve economic efficiency by using production resources, as measured by the value of the estimated production elasticity of the production function [5], also analyzed the economic and technical efficiency of marketing and butchering beef calves in Sharkia Province - Egypt. From the statistical analysis of the production function, the researchers concluded that among the problems of fattening calves are the increase in variable costs due to the high prices of feed and the deterioration of its specifications, in addition to the high prices of veterinary medicines. In a study conducted by [6] in Kut province - Iraq, with the aim of measuring the economic and technical efficiency of calf breeding projects by adopting the (DEAP) analysis method. The researchers concluded that calf breeders in the study area suffer from high feed prices and that only (14%) of the research sample achieved economic efficiency in production. The research gap appears through previous studies that researchers followed the method of relative statistical analysis of the production function, so in this research, the optimal combination of production resources will be reached, which achieves the optimal volume of production. This economic analysis depends on the (Lagrange multiplier) method, as this method provides accurate results that are useful to the producer and researcher [7]. adopted the Lagrange method in a study conducted on grape production farms, and through it, the researchers were able to reach the optimal combination of production resources and the optimal volume of production. By adopting the Lagrange multiplier method, an economic analysis will be conducted based on the specific research problem surrounding the inefficient use of production resources, which causes an increase in production costs and a decline in productivity in calf fattening farms in the study area.

Research problem: Calf fattening projects in the study area have declined due to the high production costs of these projects, and consequently the waste of productive resources.

Research objective: Conducting an economic analysis of a calf-fattening project using the estimated production function to reach the optimal combination of production resources using the Lagrange multiplier method and production resource prices with the aim of achieving optimal production.

Research hypothesis: The research is based on the hypothesis that calf-fattening projects achieve optimal production if production resources are used optimally, thus achieving economic and technical efficiency in production.

MATERIALS AND METHODS

Farm Data

After a personal interview with the manager of the calf-fattening farm in the study area, data specific to the farm under study were recorded, as shown in Table (1):

Table (1) Basic data for the calf-fattening farm in the study area for the production year 2024.

S	Items	Statement of details
1	Farm location	Muqdadiyah city - north of Diyala province
2	farm area	750 meters
3	Calf hall area	350 meters
4	Calf hall capacity	20 calves
5	Average age of calves at purchase	24-36 weeks
6	Average live weight of calf at purchase	350-400 K.g (live weight)
7	Planned fattening period	25 weeks
8	Places to buy calves	Rural areas of Diyala Province
9	Class of calves purchased	Local varieties
10	Feed store capacity	16 meters

Reference/Author based on personal interview with farm manager.

From Table (1) it is clear that the manager of the calf farm relied on his own experience in purchasing calves, as they were purchased from villages where livestock farming is widespread in the study area.

Questionnaire

A questionnaire was prepared to collect the necessary data on the calf-fattening farm in the study area.

Statistical analysis

Statistical analysis is conducted using Excel to tabulate the data, and then statistically analyzed using Eviews 12.

Production Function

It is the technical relationship between production resources and the expected production quantity in the production process. By estimating the production function, we can determine the changes that occur in the production quantity, as the multi-component production function that includes the main production resources will be estimated as follows: [8].

$$\ln Y = b_0 + b_1 \ln X_1 + b_2 \ln X_2 + b_3 \ln X_3 \dots b_n \ln X_n \dots (1)$$

Where:

Y = Quantity of production.

$X_1 \dots X_n$ = Production resources included in the production function.

$b_1 \dots b_n$ = Production resource elasticities.

Objective function

The optimal combination of productive resources refers to the quantities a producer must use to achieve technical efficiency in production and allocative efficiency in the distribution of productive resources. This can be achieved using the Lagrange multiplier method, which is used as a link between the estimated production function, after converting it to an exponential form, and the cost equation, which includes the prices of production resources. This then forms the objective function to achieve the necessary conditions for maximizing production [9].

$$C = X_1 r_1 + X_2 r_2 + X_3 r_3 + X_4 r_4 + X_5 r_5 \dots (2) \quad \text{cost equation}$$

$$C^\circ - X_1 r_1 - X_2 r_2 - X_3 r_3 - X_4 r_4 - X_5 r_5 = 0 \dots (3) \quad \text{Cost constraint equation}$$

Where:

C° = average bound production costs.

$X_1 \dots X_5$ = quantities of production resources contributing to the production process.

$r_1 \dots r_5$ = prices of production resources.

To form the objective function, the production function is transformed from its logarithmic form to its exponential form:

$$Y = b_0 X_1^{b_1} X_2^{b_2} X_3^{b_3} X_4^{b_4} X_5^{b_5} \dots \dots (4)$$

The objective function is:

$$L = b_0 X_1^{b_1} X_2^{b_2} X_3^{b_3} X_4^{b_4} X_5^{b_5} + \lambda (C^\circ - X_1 r_1 - X_2 r_2 - X_3 r_3 - X_4 r_4 - X_5 r_5) \dots (5)$$

From the objective function, the maximum limit of the production function can be found by performing the first partial differentiation for each production resource [10].

Technical efficiency of production

Through marginal analysis of the estimated production function, the marginal product, average product, and production elasticity can be determined for each productive resource contributing to the production process. These indicators can be found using the following relationships: [11].

$$\text{Marginal product: } MPP_{Xi} = \frac{\partial Y}{\partial X_i}$$

$$\text{Average product: } APP_{Xi} = \frac{Y}{X_i}$$

$$\text{Productive elasticity: } E_p = \frac{MPP_{Xi}}{APP_{Xi}}$$

Allocative efficiency

The allocative efficiency of productive resources can be determined by relying on a relationship that includes the ratio between the marginal product value of the resource and the marginal cost of the productive resource. The allocative efficiency of productive resources can be measured using the following relationship: [11].

$$r = \frac{VMP_{Xi}}{MFC_{Xi}}$$

Where:

r = allocative efficiency of resource use.

VMP_{Xi} = marginal product value of the resource.

MFC_{Xi} = marginal cost of the resource used.

RESULTS AND DISCUSSION**Investment costs**

The investment costs include all the funds invested by the producer to establish a calf-fattening farm, including the construction of the barn, feed store, and other fattening requirements, as shown in the following Table (2):

Table (2) Investment costs for the studied calf-fattening farm for the production year 2024.

S	Expenditure items	Average cost (\$)	Relative importance %
1	land rent	100.6	1.13
2	building the barn	3422.8	38.57
3	Buy feeders	604	6.81
4	Buy water basins	1006.7	11.34
5	Lighting	33.5	0.38
6	Service road	16.7	0.19
7	Feed transport vehicles	167.7	1.89
8	farm fence	167.7	1.89
9	Buying calves	3355.7	37.81
	Total	8875.4	100

Reference/Researcher based on a personal interview with the farm manager.

* Exchange rate: (1 \$ = 1490 dinars IQ.) according to the exchange rate prevailing in the Iraqi market in 2024.

It is clear from Table (2) that the highest percentage was the cost of building the calf barn, which constituted (38.57%) of the total investment costs, followed by the cost of purchasing calves, which constituted (37.81%) of the total investment costs, and then the other investment cost items come in order.

Production Resource Prices

Production resource prices are the primary determinant of production costs in agricultural projects. Therefore, production resource prices will be based on the producer's expenditures for the 2024 production year for calculating production costs, as shown in Table (3):

Table (3) Prices of production resources and production costs for the studied calf farm for the production year 2024

Production resources	The price	Unit of measurement	Quantity consumed (25 weeks)	Unit of measurement	Cost (\$/calf)/25 weeks	Contribution rate (%)
dry fodder	32	\$/ Ton	3.9	ton/ calf	127.8	17.95
green fodder	51	\$/ Ton	4.5	ton/ calf	231.8	32.56
drinking water	0.140	\$/ Liter	393.5	Liter / calf	55	7.73
working hours	3.3	\$/hour	40.4	hour / calf	135.6	19.04
Veterinary supervision costs	2.2	\$/calf	44.6	\$/calf	44.7	6.27
Maintenance costs	10.9	\$/calf	-----	-----	10.9	1.53
Transportation	44	\$/calf	-----	-----	44	6.18
Total Variable Cost	-----	\$/calf	-----	-----	649.5	
Depreciation	13	\$/calf	-----	-----	13	1.83
land rent	5	\$/calf	-----	-----	5	0.70
Interest on capital (10%)	44.5	\$/calf	-----	-----	44.2	6.21
Total fixed costs	-----	\$/calf	-----	-----	62.5	
Total costs	-----	\$/calf	-----	-----	712	100

Reference/Author based on a personal interview with the farm manager.

* Exchange rate: (1 \$ = 1490 dinars IQ.) according to the exchange rate prevailing in the Iraqi market in 2024.

It is clear from Table (3) that the costs of green fodder constituted the largest proportion of the total production costs, amounting to (32.56%), followed by the cost of working hours, which constituted (19.04%) of the total production costs, followed by other production cost items.

Statistical analysis

A statistical analysis was conducted on the multivariate production function, which included production resources (dry feed, green feed, drinking water, working hours, and veterinary supervision expenses), as shown in the following Table (4):

Table (4) Statistical analysis of the multivariate production function for the calf-fattening farm for the production year 2024

Dependent Variable: LOG(Y)

Method: Least Squares

Date: 03/14/25 Time: 22:59

Sample: 1 25

Included observations: 25

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.467872	0.255443	5.746375	0.0000
LOG(X1)	0.074209	0.041893	1.771373	0.0925
LOG(X2)	0.632972	0.186443	3.394993	0.0030
LOG(X3)	0.301158	0.103493	2.909942	0.0090
LOG(X4)	0.166488	0.107293	1.551709	0.1372
LOG(X5)	0.350328	0.127452	2.748702	0.0128
R-squared	0.982716	Mean dependent var		1.750464
Adjusted R-squared	0.978168	S.D. dependent var		0.112146
S.E. of regression	0.016570	Akaike info criterion		-5.156852
Sum squared resid	0.005217	Schwarz criterion		-4.864321
Log likelihood	70.46065	Hannan-Quinn criter.		-5.075716
F-statistic	216.0615	Durbin-Watson stat		2.280326
Prob(F-statistic)	0.000000			

The statistical analysis in Table (4) showed the significance of each of the variables (X_1 , X_2 , X_3 , and X_5) through the value of the (t) test at a significance level of (5%), while the significance of the variable (X_4) is proven at a significance level of (10%). Moreover, the significance of the function as a whole is proven through the value of the (F) test at a significance level of (5%). In addition, from the value of the coefficient of determination (R^2) it became clear that (98%) of the changes occurring in production are caused by the production resources included in the estimated function. The calculated value of the Durbin-Watson test was (2.2), which means that there is no problem of autocorrelation based on the rule ($4 - d_u > d^* > d_u$) and at a significance level of (5%) and the number of variables ($K=5$). To ensure that the estimated model is free from the problem of heteroscedasticity, the Park test was conducted by performing a multiple regression of the logarithm of the least squares residuals as a dependent factor and the logarithm of production resources as independent factors, as in the following Table (5):

Table (5) Park test for the multivariate production function of the calf-fattening farm for the production year 2024.

Dependent Variable: LOG(EISQ)				
Method: Least Squares				
Date: 03/14/25 Time: 23:42				
Sample: 1 25				
Included observations: 25				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.410648	13.28392	0.030913	0.9757
LOG(X1)	-0.010355	2.178593	-0.004753	0.9963
LOG(X2)	-4.308625	9.695665	-0.444387	0.6618
LOG(X3)	2.009146	5.381975	0.373310	0.7131
LOG(X4)	3.348802	5.579609	0.600186	0.5555
LOG(X5)	-5.490100	6.627954	-0.828325	0.4178
R-squared	0.057935	Mean dependent var		-0.158676
Adjusted R-squared	-0.189977	S.D. dependent var		0.789936
S.E. of regression	0.861711	Akaike info criterion		2.745768
Sum squared resid	14.10836	Schwarz criterion		3.038299
Log likelihood	-28.32211	Hannan-Quinn criter.		2.826904
F-statistic	0.233691	Durbin-Watson stat		2.167240
Prob(F-statistic)	0.942899			

It is clear from Table (5) that the values of the (t) test are not significant for all variables of the estimated model and at different significance levels. This confirms that the estimated model is free from the problem of heteroscedasticity of variance, in addition to the lack of significance of the function as a whole through the value of the (F) test.

Economic analysis

Optimal Product Behavior

The optimal product behavior can be determined by the optimal combination of production resources that achieves optimal production. This can be arrived at mathematically by formulating the objective function and then conducting marginal analysis to arrive at the technical efficiency of production.

Production resource prices and estimated production cost levels:

$$C^0 = 712 \text{ \$/calf}$$

$$PX_1 = \text{Price of dry feed} = 32 \text{ \$/ton}$$

$$PX_2 = \text{Price of green feed} = 51 \text{ \$/ton}$$

$$PX_3 = \text{Price of drinking water} = 0.140 \text{ \$/liter}$$

$$PX_4 = \text{Price of labor per hour} = 3.3 \text{ \$/hour}$$

$$PX_5 = \text{Veterinary supervision expenses} = 2.2 \text{ \$/ calf}$$

Depending on the prices of production resources and the average cost of the calf in the research sample, the (cost constraint) can be formed as follows:

$$712 - 32(X_1) - 51(X_2) - 0.140(X_3) - 3.3(X_4) - 2.2(X_5) = 0 \dots (6)$$

The double logit model is chosen because it passed the statistical tests after analysis using (OLS) method; the production function included the following variables:

Y = Average production quantity (K.g/calf)

X₁ = Average dry feed quantity (tons/calf)

X₂ = Average green feed quantity (tons/calf)

X₃ = Average drinking water quantity (liters/calf)

X₄ = Average labor hours (hours/calf)

X₅ = Average veterinary expenses (\$/calf)

$$\ln(Y) = 1.467 + 0.0742 \ln(X_1) + 0.632 \ln(X_2) + 0.301 \ln(X_3) + 0.166 \ln(X_4) + 0.350 \ln(X_5)$$

Convert the function to exponential form:

$$Y = 4.336 X_1^{0.0742} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{0.350} \dots (7)$$

To form the objective function, we can adopt Equation (6) and Equation (7) and use the Lagrange multiplier:

$$L = (4.336 X_1^{0.0742} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{0.350}) + \lambda(712 - 32(X_1) - 51(X_2) - 0.140(X_3) - 3.3(X_4) - 2.2(X_5) = 0 \dots (8)$$

Based on (Equation 8), the partial differentiation of the production resources included in the estimated function and the Lagrange multiplier can be performed as follows:

$$\begin{aligned} \frac{\partial L}{\partial X_1} &= (4.336(0.0742)X_1^{0.0742-1}X_2^{0.632}X_3^{0.301}X_4^{0.166}X_5^{0.350}) + \lambda(712 - 32(X_1) - 51(X_2) - 0.140(X_3) \\ &\quad - 3.3(X_4) - 2.2(X_5) = 0 \\ &= (0.321)X_1^{-0.925}X_2^{0.632}X_3^{0.301}X_4^{0.166}X_5^{0.350}/32 = \lambda \\ &= (0.01003125)X_1^{-0.925}X_2^{0.632}X_3^{0.301}X_4^{0.166}X_5^{0.350} = \lambda \dots (9) \end{aligned}$$

In the same way, we derive the other variables of the production function, which are:

$$\frac{\partial L}{\partial X_2} = (0.0272912)X_1^{0.0742}X_2^{-0.368}X_3^{0.301}X_4^{0.166}X_5^{0.350} = \lambda \dots (10)$$

$$\frac{\partial L}{\partial X_3} = (9.3224)X_1^{0.0742}X_2^{0.632}X_3^{-0.699}X_4^{0.166}X_5^{0.350} = \lambda \dots (11)$$

$$\frac{\partial L}{\partial X_4} = (0.21811393)X_1^{0.0742}X_2^{0.632}X_3^{0.301}X_4^{-0.834}X_5^{0.350} = \lambda \dots (12)$$

$$\frac{\partial L}{\partial X_5} = (0.6898181)X_1^{0.0742}X_2^{0.632}X_3^{0.301}X_4^{0.166}X_5^{-0.650} = \lambda \dots (13)$$

$$\frac{\partial L}{\partial \lambda} = (712 - 32(X_1) - 51(X_2) - 0.140(X_3) - 3.3(X_4) - 2.2(X_5) = 0 \dots (14)$$

To find the values of resources (X₂, X₃, X₄, and X₅), equations (10, 11, 12, and 13) are divided by equation (9). Since the Lagrange multiplier (λ) has a value of one, we can briefly find the values of production resources in terms of resource (X₁) by performing the following mathematical operations:

$$\frac{0.0272912 X_1^{0.0742} X_2^{-0.368} X_3^{0.301} X_4^{0.166} X_5^{0.350}}{0.01003125 X_1^{-0.925} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{0.350}} = \frac{\lambda}{\lambda} = 1$$

$$X_2 = 2.7206 X_1 \dots (15)$$

$$\frac{9.3224 X_1^{0.0742} X_2^{0.632} X_3^{-0.699} X_4^{0.166} X_5^{0.350}}{0.01003125 X_1^{-0.925} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{0.350}} = \frac{\lambda}{\lambda} = 1$$

$$X_3 = 929.335 X_1 \dots (16)$$

$$\frac{0.21811393 X_1^{0.0742} X_2^{0.632} X_3^{0.301} X_4^{-0.834} X_5^{0.350}}{0.01003125 X_1^{-0.925} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{0.350}} = \frac{\lambda}{\lambda} = 1$$

$$X_4 = 21.743 X_1 \dots \dots (17)$$

$$\frac{0.6898181 X_1^{0.0742} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{-0.650}}{0.01003125 X_1^{-0.925} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{0.350}} = \frac{\lambda}{\lambda} = 1$$

$$X_5 = 68.766 X_1 \dots \dots (18)$$

To find the value of resource (X_1), we substitute equations (15), (16), (17), (18) into equation (14) as follows:

$$(712 - 32(X_1) - 51 (2.7206 X_1) - 0.140 (929.335 X_1) - 3.3(21.743 X_1) - 2.2 (68.766 X_1) = 0$$

$$712 - 459.8926 X_1$$

$$X_1 = \frac{712}{459.8926} = 1.5 \quad \text{Ton/calf within 25 weeks}$$

To find the quantities of other resources, we substitute the value of (X_1) into equations (15) (16) (17) (18) as follows:

$$X_2 = 2.7206 (1.548) = 4.2 \quad \text{Ton/calf within 25 weeks}$$

$$X_3 = 929.335 (1.548) = 1438.6 \quad \text{Liter/calf within 25 weeks}$$

$$X_4 = 21.743 (1.548) = 33.6 \quad \text{Working Hour/calf within 25 weeks}$$

$$X_5 = 68.766 (1.548) = 106.4 \quad \text{\$/calf within 25 weeks}$$

By substituting the optimal quantities of production resources in Equation (7), we obtain the optimal production, which represents the best level of productivity that can be achieved for one calf within 25 weeks, as follows:

$$Y = 4.336 X_1^{0.0742} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{0.350}$$

$$Y = 4.336 (1.5)^{0.0742} (4.2)^{0.632} (1438.6)^{0.301} (33.6)^{0.166} (106.4)^{0.350}$$

$$Y = 4.336 (1.03)(2.4)(8.9)(1.7)(5.1) = 827 \text{ K. G}$$

Where a weight of (827) K.g can be achieved for each calf by adopting the optimal combination of resources that was reached within the specified cost constraint of (712) \$ for the calf during the fattening period of (25) weeks.

Table 6. Comparison of the combination of resources actually used with the optimal combination for the studied calf farm.

Production resources	The combination actually used	The optimal combination	Unit of measurement
X_1	3.9	1.5	ton/calf
X_2	4.5	4.2	ton/calf
X_3	393.5	1438.6	Liter/calf
X_4	40.4	33.6	hour/calf
X_5	44.6	106.4	\\$/calf
Productivity (Y)	369	827	K. g/calf

Reference/Author based on the questionnaire and the estimated production function.

It is clear from Table (6) that there is a difference between the quantities of inputs actually used and the quantities of optimal resources that achieve optimal production, as well as the average actual productivity, which amounted to (369) K.g/calf, while the optimal productivity amounted to (827) K.g/

calf during the fattening period of (25) weeks.

Measuring Production Efficiency

Technical production efficiency can be achieved when the highest possible productivity is achieved using the same amount of available production resources. Technical efficiency is a concept that refers to the relationship between production resources and production. A project is most efficient if it achieves the highest possible productivity using the same number of resources [13]. Calculating technical efficiency requires applying the condition of the law of diminishing returns, which stipulates that the quantities of productive resources should be fixed at their arithmetic mean, increasing the quantity of one productive resource. Then calculating the marginal product and the average product of the resource by performing partial differentiation on the production function estimated by adopting the average quantities of productive resources actually used and fixed at the arithmetic mean, as follows:

$$\overline{X}_1 = 3.9 \text{ tons/ calf}$$

$$\overline{X}_2 = 4.5 \text{ tons/calf}$$

$$\overline{X}_3 = 393.3 \text{ liter/ calf}$$

$$\overline{X}_4 = 40.4 \text{ hour/ calf}$$

$$\overline{X}_5 = 44.6 \text{ \$/ calf}$$

To find the marginal product of one of the productive resources, we substitute the quantities of the combination of resources into the estimated production function, except for the variable productive resource, which is substituted into the optimal quantity that was reached to determine whether it achieves technical efficiency or not, as follows:

$$Y = 4.336 X_1^{0.0742} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{0.350}$$

$$MP_{X_1} = \frac{\partial Y}{\partial X_1} = (4.336)(0.0742) X_1^{0.0742-1} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{0.350}$$

$$MP_{X_1} = (0.321) X_1^{-0.925} (4.4)^{0.632} (393.3)^{0.301} (40.4)^{0.166} (44.6)^{0.350}$$

$$MP_{X_1} = (0.321) X_1^{-0.925} (2.550) (6.039) (1.847) (3.778)$$

$$MPP_{X_1} = \frac{(34.493)}{(1.5)^{0.925}}$$

$$MPP_{X_1} = \frac{(34.493)}{(1.455)} = 23.7 \text{ K.G Marginal product for } (X_1)$$

In the same way, we derive the other variables of the production function, which are:

$$MP_{X_2} = \frac{\partial Y}{\partial X_2} = (4.336)(0.632) X_1^{0.0742} X_2^{0.632-1} X_3^{0.301} X_4^{0.166} X_5^{0.350}$$

$$MP_{X_2} = (2.7403) (3.9)^{0.0742} (X_2)^{-0.368} (393.3)^{0.301} (40.4)^{0.166} (44.6)^{0.350}$$

$$MPP_{X_2} = \frac{(127.716)}{1.695} = 75.3 \text{ K.G Marginal product for } (X_2)$$

$$MP_{X_3} = \frac{\partial Y}{\partial X_3} = (4.336)(0.301) X_1^{0.0742} X_2^{0.632} X_3^{0.301-1} X_4^{0.166} X_5^{0.350}$$

$$MP_{X_3} = (1.305) (3.9)^{0.0742} (4.5)^{0.632} (X_3)^{-0.699} (40.4)^{0.166} (44.6)^{0.350}$$

$$MPP_{X_3} = \frac{(23.901)}{(1438.6)^{0.699}} = 0.148 \text{ K.G Marginal product for } (X_3)$$

$$MP_{X_4} = \frac{\partial Y}{\partial X_4} = (4.336)(0.166) X_1^{0.0742} X_2^{0.632} X_3^{0.301} X_4^{0.166-1} X_5^{0.350}$$

$$MP_{X_4} = (0.719) (3.9)^{0.0742} (4.5)^{0.632} (393.3)^{0.301} (X_4)^{-0.834} (44.6)^{0.350}$$

$$MPP_{X_4} = \frac{(44.180)}{(33.6)^{0.834}} = 2.3 \text{ K.G Marginal product for } (X_4)$$

$$MP_{X_5} = \frac{\partial Y}{\partial X_5} = (4.336)(0.350) X_1^{0.0742} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{0.350-1}$$

$$MP_{X_5} = (1.517)(3.9)^{0.0742} (4.5)^{0.632} (393.3)^{0.301} (40.4)^{0.166} (X_5)^{-0.650}$$

$$MPP_{X_5} = \frac{(46.531)}{(106.4)^{0.650}} = 2.2 \text{ K.G Marginal product for } (X_5)$$

To find the average production of productive resources, we can divide (Y) by (Xi) separately as follows:

$$APP_{X_1} = \frac{Y}{X_1} = 4.336 X_1^{0.0742-1} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{0.350}$$

$$APP_{X_1} = 4.336 X_1^{-0.925} (4.5)^{0.632} (393.3)^{0.301} (40.4)^{0.166} (44.6)^{0.350}$$

$$APP_{X_1} = 4.336 X_1^{-0.925} (2.5)(6.03)(1.8)(3.7)$$

$$APP_{X_1} = \frac{(435.332)}{(1.5)^{0.925}} = 299.1 \text{ K.G Average product for } (X_1)$$

In the same way, we derive the other variables of the production function, which are:

$$APP_{X_2} = \frac{Y}{X_2} = 4.336 X_1^{0.0742} X_2^{0.632-1} X_3^{0.301} X_4^{0.166} X_5^{0.350}$$

$$APP_{X_2} = 4.336 (3.9)^{0.0742} X_2^{-0.368} (393.3)^{0.301} (40.4)^{0.166} (44.6)^{0.350}$$

$$APP_{X_2} = \frac{(191.546)}{(4.2)^{0.368}} = 113 \text{ K.G Average product for } (X_2)$$

$$APP_{X_3} = \frac{Y}{X_3} = 4.336 X_1^{0.0742} X_2^{0.632} X_3^{0.301-1} X_4^{0.166} X_5^{0.350}$$

$$APP_{X_3} = 4.336 (3.9)^{0.0742} (4.5)^{0.632} X_3^{-0.699} (40.4)^{0.166} (44.6)^{0.350}$$

$$APP_{X_3} = \frac{(79.413)}{(438.6)^{0.699}} = 1.1 \text{ K.G Average product for } (X_3)$$

$$APP_{X_4} = \frac{Y}{X_4} = 4.336 X_1^{0.0742} X_2^{0.632} X_3^{0.301} X_4^{0.166-1} X_5^{0.350}$$

$$APP_{X_4} = 4.336 (3.9)^{0.0742} (4.5)^{0.632} (393.3)^{0.301} X_4^{-0.834} (44.6)^{0.350}$$

$$APP_{X_4} = \frac{(264.712)}{(33.6)^{0.834}} = 14.1 \text{ K.G Average product for } (X_4)$$

$$APP_{X_5} = \frac{Y}{X_5} = 4.336 X_1^{0.0742} X_2^{0.632} X_3^{0.301} X_4^{0.166} X_5^{0.350-1}$$

$$APP_{X_5} = 4.336 (3.9)^{0.0742} (4.5)^{0.632} (393.3)^{0.301} (40.4)^{0.166} X_5^{-0.650}$$

$$APP_{X_5} = \frac{(476.483)}{(106.4)^{0.650}} = 22.9 \text{ K.G Average product for } (X_5)$$

Table (7) Quantities of marginal product and average product of production resources in the studied calf farm for the production year 2024.

Variable production resource	Combination of resources actually used	The optimal combination of productive resources	Marginal product (K.g/ calf)	Average product (K.g/calf)	Elasticity Production
X ₁	3.9	1.5	23.7	299.1	0.079
X ₂	4.5	4.2	75.3	113	0.666
X ₃	393.3	438.6	0.148	1.1	0.134
X ₄	40.4	33.6	2.3	14.1	0.163
X ₅	44.6	106.4	2.2	22.9	0.096

It is clear from Table (7) that the average product of all productive resources is higher than the marginal product. This shows that the production process is in the second stage of the law of diminishing returns, which is the economic stage. This is confirmed by the value of production elasticity being less than one and positive. This explains that the optimal quantities of productive resources achieve technical efficiency in production.

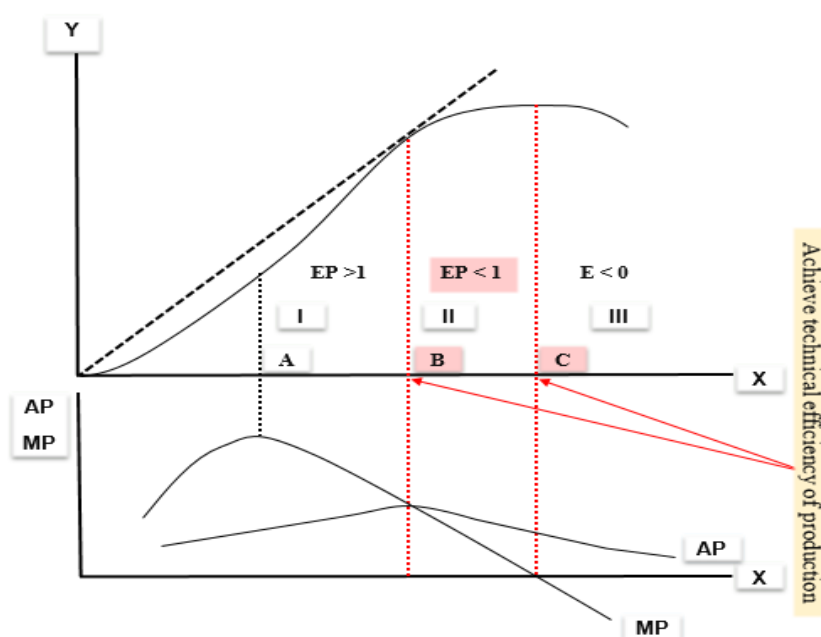


Fig. 1 Technical efficiency of production achieved in the second stage of the law of diminishing return

From the figure, (1) it appears that the technical efficiency of production is achieved between points (B) and (C) where the average product is higher than the marginal product and points (B) and (C) determine the economic stage of the law of diminishing returns and since this stage achieves the highest possible productivity. It is not possible to increase production resources after point (C) because the production process enters the stage of absolute diminishing returns due to the control of short-term conditions over the production process, as the farm area cannot be increased.

Production resource efficiency

This type of efficiency reflects the farm manager's ability to optimally utilize production resources. The project is more cost-efficient in selecting the optimal combination of production resources. The allocative efficiency of using productive resources is achieved when the value of the ratio of the marginal product of the productive resource to the marginal cost of that resource approaches one. If the value of this ratio is less than one, this means that the marginal cost is higher than the achieved return, which means that there is waste in using the available production resources, which requires their redistribution. If the value is greater than one, this means that the achieved return is greater than the marginal cost added, which provides an opportunity to increase profits. The allocative efficiency of productive resources can be measured by the following relationship: [11].

$$r = \frac{VMP_{xi}}{MFC_{xi}}$$

Where:

r = allocative efficiency of resource use.

VMP_{xi} = marginal product value of the resource.

MFC_{xi} = marginal cost of the productive resource.

Also:

$r = 1$: Efficient use of productive resources.

$r < 1$: More productive resources can be added.

$r > 1$: Wasteful use of resources requires redeployment.

The allocative efficiency of production resources can be reached by finding the value of (VMPxi) based on Table (7) as shown in the following Table (8):

Table (8) Allocative efficiency of production resources for the studied calf-fattening farm for the production year 2024.

Xi	Marginal product per resource (K.g/calf)	Product selling price (\$/K.g)	VMPxi (\$/K.g)	MFCxi (\$/per unit added)	Allocative efficiency of resource (r)
X ₁	23.7	7.5	7.5	32	5.8
X ₂	75.3	7.5	7.5	51	7.2
X ₃	0.148	7.5	7.5	0.140	0.02
X ₄	2.3	7.5	7.5	3.3	0.44
X ₅	2.2	7.5	7.5	2.2	0.29

Reference/Author based on a personal interview with the farm manager, Table (7).

* Exchange rate: (1 \$ = 1490 dinars IQ.) according to the exchange rate prevailing in the Iraqi market in 2024.

Table (8) shows that the value of the allocative efficiency of the two resources (X₁), which represents dry fodder, and (X₂), which represents green fodder, is greater than one. This means that there is waste in the use of dry and green fodder according to the (Debertin) formula, which requires reorganizing the use of these two resources. This result is consistent with [14], who confirmed that one of the drawbacks of calf fattening projects is their sensitivity to fodder purchase prices. As for the allocative efficiency of other production resources, it is less than one, which means that there is an opportunity to increase the use of these resources as needed in order to achieve additional profits.

CONCLUSION

After an economic analysis of the studied calf-fattening farm, it became clear that the producer was unable to achieve the optimal combination of production resources, as demonstrated by the difference between the actual quantities of production resources used and the optimal quantities achieved. The optimal productivity reached (827) kg/calf, which was achieved through the production function estimated by applying the optimal combination of production resources, while the average actual productivity was (369) kg/calf. It was also found that the product could operate in the second stage of the law of diminishing returns when applying the optimal combination of production resources, as the production elasticity of production resources reached less than one and with a positive sign. In addition, it was found that there was waste in using dry fodder and green fodder, as the allocative efficiency of these two resources was greater than one, while it was less than one for the resources (drinking water, working hours, veterinary supervision costs). We conclude from this that the producer can reduce the use of fodder according to certain scientific recommendations and increase the use of drinking water, working hours, and veterinary supervision costs to achieve complete allocative efficiency.

Recommendation

1. The effective role of agricultural guidance in educating calf breeders about applying the results of applied scientific research that provides calf breeders with a roadmap for using production resources with the aim of reducing waste by using production resources and achieving efficiency.
2. Providing sufficient information on the prices of production requirements, including imported dry feed and locally produced dry feed, to enable calf farmers to purchase feed at reasonable prices, which helps reduce production costs.

3. Supporting the cultivation of green fodder for calf breeders by providing fodder seeds and the necessary fertilizers for fodder cultivation by the Iraqi Ministry of Agriculture.
4. Finding suitable markets for calf farm products, including by-products such as hides, and encouraging the use of manure as organic fertilizer to maximize the benefits of calf fattening farm products.
5. Providing guidance on technical operations in calf fattening farms, including improving the necessary veterinary services, in addition to how to provide a suitable environment for housing calves in pens.

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